Contents

Overview 3

Chapter 1: Basic Skills

1.1 Graphing Systems of Linear Equations 6
1.2 Exponent Rules and Polynomials 10
1.3 Factorization 14
1.4 Simple Quadratic Equations 17
1.5 Triangles and Trigonometric Ratios 19

Chapter 2: Equivalent Algebraic Expressions

2.1 Adding and Subtracting Polynomials 22
2.2 Multiplying and Factoring Polynomials 26
2.3 Multiplying and Dividing Rational Expressions 29
2.4 Simplifying Rational Expressions 33

Chapter 3: Quadratic Functions

3.1 The Domain and Range of a Function 38
3.2 Function Notation 43
3.3 Maximum or Minimum Values of Quadratic Functions 46
3.4 Operations with Radicals 50
3.5 Solving Quadratic Equations 53
3.6 Families of Quadratic Functions 57
3.7 Solving Linear-quadratic Systems 60
3.8 Transformations 64
3.9 The Inverse of a Function 70

Chapter 4: Exponential Functions

4.1 Exponents 74
4.2 Rational Exponents 77
4.3 Algebraic Expressions Involving Exponents 80
4.4 Exponential Functions 83
4.5 Transformations of Exponential Functions 86
4.6 Applications of Exponential Functions 92

SAMPLE

ISBN: 978-1-77149-222-5

Chapter 5: Trigonometry

5.1 Reciprocal Trigonometric Ratios 96
5.2 Special Angles 99
5.3 Angles in the Cartesian Plane 102
5.4 Trigonometric Identities 106
5.5 The Sine Law and Cosine Law 109
5.6 Solving 3-D Problems Using Trigonometry 113

Chapter 6: Sinusoidal Functions

6.1 Properties of Periodic Functions 116
6.2 Properties of Sinusoidal Functions 120
6.3 Transformations of Sinusoidal Functions 125
6.4 Graphing and Modelling 129
6.5 Applications of Sinusoidal Models 133

Chapter 7: Discrete Functions

7.1 Discrete Functions and Sequences 136
7.2 Recursive Procedures 140
7.3 Pascal's Triangle and Expanding Binomial Powers 143
7.4 Arithmetic Sequences 146
7.5 Geometric Sequences 150
7.6 Arithmetic Series 154
7.7 Geometric Series 157

Chapter 8: Financial Applications

8.1 Simple Interest 160
8.2 Compound Interest 163
8.3 Present Value 166
8.4 Annuities 169
8.5 Present Value of an Annuity 172

Cumulative Review 177
Handy Reference 191
Answers 197

SAMPLE

ISBN: 978-1-77149-222-5

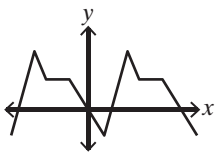
6 Sinusoidal Functions

Words to Learn

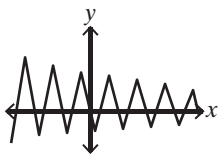
Periodic function:

a function that repeats its values in regular intervals

e.g.



periodic

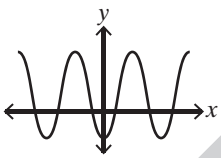


nonperiodic

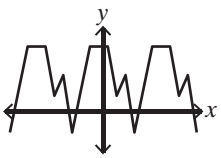
Sinusoidal function:

a periodic function that resembles a smooth curve that is symmetrical

e.g.



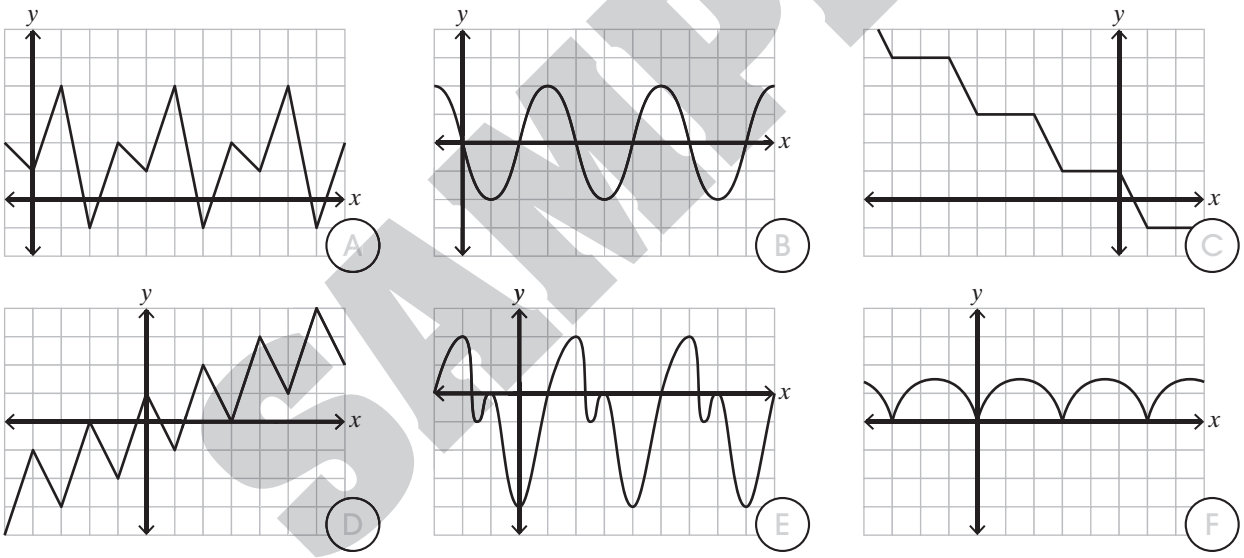
periodic and sinusoidal



periodic but not sinusoidal

6.1 Properties of Periodic Functions

Identify and check the graphs that are periodic. Then answer the question.

① 

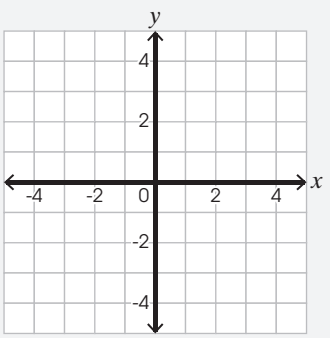
② Which table represents the values of a periodic function? Explain your choice. Then graph the function.

A

x	-5	-4	-3	-2	-1	0	1	2	3	4
y	2	0	-2	4	2	0	-2	4	2	0

B

x	-5	-4	-3	-2	-1	0	1	2	3	4
y	4	3	2	1	0	-1	-2	-3	-4	-5

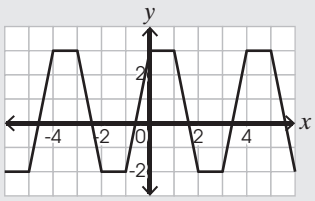


SAMPLE

ISBN: 978-1-77149-222-5

Example

Determine the key features of the periodic graph.



Solution:

period: 4 ← The graph repeats its cycle every 4 units on the x-axis.

peak: 3 ← the maximum y-value

trough: -2 ← the minimum y-value

range: $\{y \in \mathbb{R} \mid 3 \leq y \leq -2\}$ ← all possible values of y

equation of the axis: $y = \frac{3 + (-2)}{2}$

$y = 0.5$

amplitude: $\frac{3 - (-2)}{2} = 2.5$ ← can also be determined using the function's axis

$3 - 0.5 = 2.5$ or $0.5 - (-2) = 2.5$

Try This

period: _____

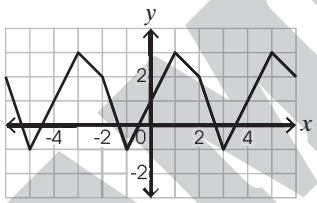
peak: _____

trough: _____

range: _____

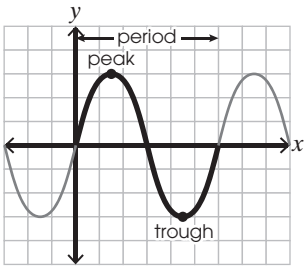
equation of the axis: _____

amplitude: _____



Hint

Key Features of Periodic Functions



- **period:** the horizontal distance needed for the graph of a periodic function to complete one cycle
- **peak:** the maximum point on a graph
- **trough:** the minimum point on a graph
- **range:** the set of all y-values
- **equation of the axis:** the equation of the horizontal line halfway between the maximum and minimum values

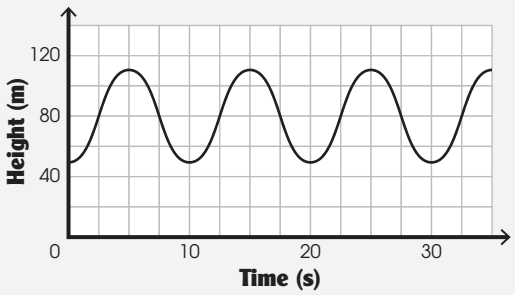
$$y = \frac{\text{max. value} + \text{min. value}}{2}$$

- **amplitude:** half the difference between the maximum and minimum values; or the vertical distance from the function's axis to the maximum or minimum value

Determine the key features of the periodic graphs.

③

Height of a Blade on a Wind Turbine



period: _____

peak: _____

trough: _____

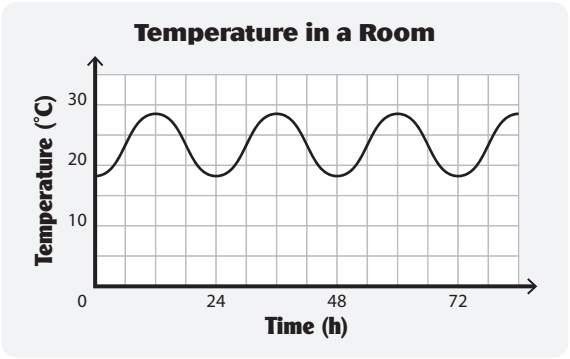
amplitude: _____

range: _____

equation of the axis: _____

SAMPLE

④



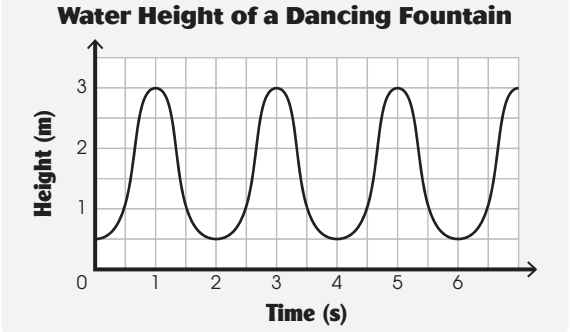
period: _____ range:

peak: _____ _____

trough: _____ equation of the axis:

amplitude: _____ _____

⑤



period: _____ range:

peak: _____ _____

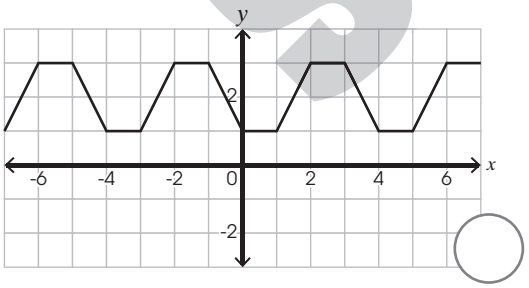
trough: _____ equation of the axis:

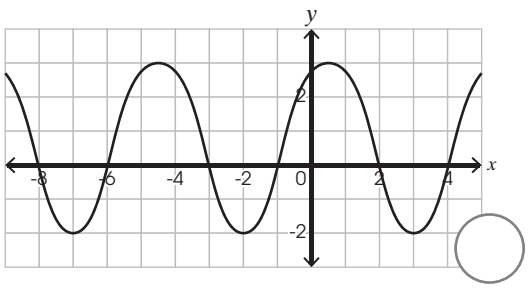
amplitude: _____ _____

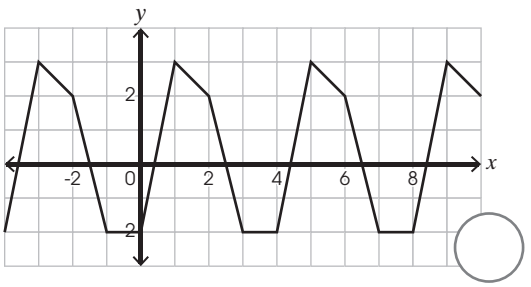
Complete the table. Then match the graphs.

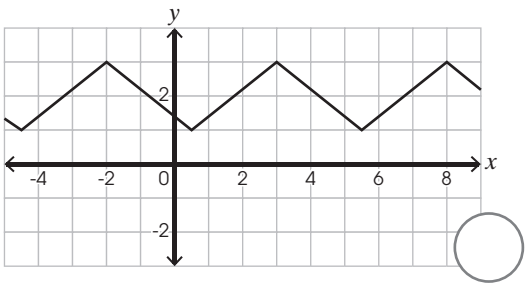
⑥

Graph	A	B	C	D
period	4	4	5	5
peak	3	3	3	3
amplitude	_____	1	_____	2.5
equation of the axis	$y = 0.5$	_____	$y = 2$	_____









SAMPLE

ISBN: 978-1-77149-222-5

Determine whether each scenario will produce a periodic or a nonperiodic graph. For the periodic graphs, determine the independent and dependent variables.

⑦ May is skipping with a jump rope.

_____

- independent variable: _____
- dependent variable: _____

⑧ The water level of a bay is changing due to tides.

_____

- independent variable: _____
- dependent variable: _____

⑨ Kobe is swimming and he wants to find out the distance he has swum.

_____

- independent variable: _____
- dependent variable: _____

An independent variable is a variable whose values are chosen and is usually presented on the x-axis.

A dependent variable is a variable whose values are calculated and is usually presented on the y-axis.

e.g. Jody is jogging.

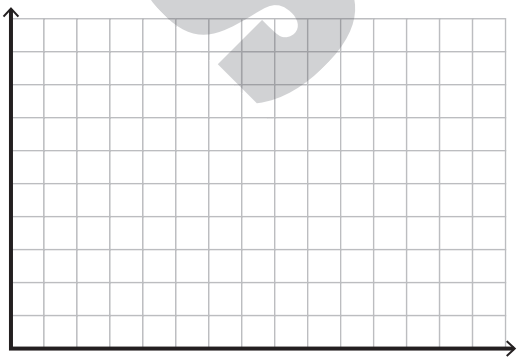
- *independent variable: time*
- *dependent variable: distance jogged*

A table of values is given for each scenario. Plot the points and answer the question.

⑩ Water is pumped into and removed from a tube. The table records the amount of water in the tube at specific times.

A

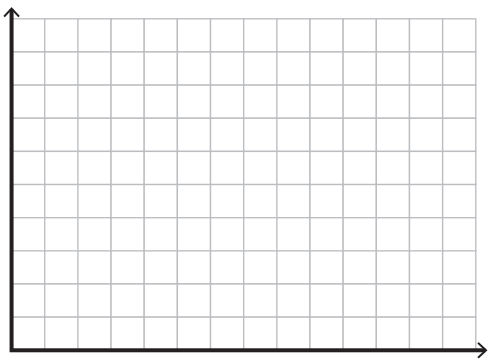
Time (min)	0	1	2	3	4	5	6	7
Amount (mL)	200	250	250	190	200	250	250	200



Joshua is on a Ferris wheel. His height above the ground over time is recorded in the table.

B

Time (s)	0	15	30	45	60	75	90
Height (m)	2	8	2	8	2	8	2



⑪ Which graph is periodic? Find its period, peak, trough, range, equation of the axis, and amplitude.

_____

_____

SAMPLE

ISBN: 978-1-77149-222-5